## Rapid Note

## Fundamental constraints for the mechanism of superconductivity in cuprates

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**Abstract.** Considerable progress has been made over the last decade in understanding the phenomenological properties of the cuprate high- $T_c$  superconductors and in producing well characterized high quality materials. Nevertheless, the pairing mechanism itself remains controversial. We establish a criterion to test theories for layered superconductors relying on a substantial interlayer contribution. The criterion is based on the ratio of the interlayer contribution to the total superfluid density, which is traced back to the inverse squared effective mass anisotropy,  $1/(1 + 2\gamma^2)$ .  $\gamma$  can be measured rather accurately by various experimental techniques. It turns out that models relying on interlayer pairing cannot be considered as serious candidates for the mechanism of superconductivity in cuprate superconductors.

**PACS.** 74.20.-z Theories and models of superconducting state – 74.20.Mn Nonconventional mechanisms (spin fluctuations, polarons and bipolarons, resonating valence bond model, anyon mechanism, marginal Fermi liquid, Luttinger liquid, etc.)

One candidate mechanism to explain superconductivity in the cuprates is the interlayer tunneling (ILT) model proposed by Anderson and coworkers [1–3]. There, superconductivity is supposed to result primarily from an interlayer coupling mechanism. It has been argued [3,4], that the comparison of the measured interlayer magnetic penetration depth  $\lambda_c$  with the value determined from the ILT-model condensation energy,  $\lambda_c^{ILT}$ , is a crucial test (cdenotes the c-axis of the unit cell). Recent direct measurements of  $\lambda_c$  in Tl<sub>2</sub>Ba<sub>2</sub>CuO<sub>6+ $\delta$ </sub> [5,6] and HgBa<sub>2</sub>CuO<sub>4+ $\delta$ </sub> [7] make it unlikely that the present version of the ILT model is a serious candidate for the mechanism of superconductivity in cuprate superconductors. Indeed,  $\lambda_c$  turns out to be much larger than  $\lambda_c^{ILT}$  [3–7]. However, it is important to recognize, that this approach is only applicable to theories, which can provide an estimate for  $\lambda_c$ .

Here we introduce a more general measure for the ratio between the interlayer – and total pairing interaction in the superconducting state. For this purpose the system is subjected to phase twists  $k_i$  along three respective crystallographic axes i = a, b, c. In the presence of such a phase twist and in the limit  $k_i \rightarrow 0$ , the free energy density then reads as (see *e.g.* [8,9])

$$f_i = \frac{k_B T}{2} \Upsilon_i k_i^2. \tag{1}$$

The helicity modulus  $\Upsilon_i$  is given as

$$\Upsilon_i = \frac{\varPhi_0^2}{16\pi^3 \lambda_i^2},\tag{2}$$

$$\frac{1}{\lambda_i^2} = \frac{16\pi^3 \hbar^2 n_s}{M_i \Phi_o^2} \,. \tag{3}$$

 $\Phi_0$  is the flux quantum,  $\lambda_i$  the magnetic penetration depth,  $M_i$  denotes the effective pair mass appearing in the gradient term of an anisotropic Ginzburg-Landau action and  $n_s$  is the superfluid number density. Imposing such twists of magnitude  $|k_i|$  along the directions i = a, b, c, respectively, the ratio

$$\eta = \frac{f_c}{f_a + f_b + f_c} = \frac{\Upsilon_c}{\Upsilon_a + \Upsilon_b + \Upsilon_c}$$
$$= 1/\left(\left(\lambda_c^2 \left(1/\lambda_a^2 + 1/\lambda_b^2 + 1/\lambda_c^2\right)\right)\right)$$
(4)

measures the fraction which the interlayer coupling contributes to the total free energy density of the superfluid. For tetragonal systems, where  $\lambda_a = \lambda_b \equiv \lambda_{\parallel}, \ \lambda_c \equiv \lambda_{\perp}$ , it reduces to the simple form

$$\eta = \frac{1}{1+2\gamma^2} \quad , \quad \gamma = \sqrt{\frac{M_\perp}{M_\parallel}}$$
 (5)

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**Table 1.** Various experimental estimates for  $\gamma$  and  $\eta$ .

	$T_c$ [K]	$\gamma$	$\eta$	Source
$YBa_2Cu_3O_{7-\delta}$	91.7	8.95	0.006	[10, 13]
$La_{2-x}Sr_xCuO_4$	35	14.0	0.0025	[14]
$\mathrm{HgBa_2CuO_{4.1}}$	94.1	26.7	0.0007	[11]
${\rm HgBa_2Ca_2Cu_3O_{8+\delta}}$	133	52	0.0002	[15]
$\mathrm{Tl_2Ba_2CuO_{6+\delta}}$	87.6	117	0.00004	[16]
$NbSe_2$	7	3	0.053	[17]

 $\gamma$  is the anisotropy parameter, which can be measured by various experimental techniques.  $\eta = 1/3$  corresponds to the case where the pairing interaction supplies the same fraction to the superfluid in the *a*-, *b*- and *c*-direction, while in a two-dimensional superconductor,  $\eta = 0$ .

In Table 1 we list some experimental estimates for cuprate superconductors close to optimum doping and – for comparison – we included the conventional layered superconductor NbSe<sub>2</sub>.

Noting that optimally doped YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> is the most isotropic cuprate and  $\gamma$  is known to increase by approaching the underdoped limit [10–17], the listed  $\eta$  values clearly reveal that superfluidity in cuprate superconductors is nearly two – dimensional. Consequently, the small interlayer pairing contribution to the superfluid makes it unlikely that theoretical models relying on a significant interlayer pairing contribution, such as the ILT model, are serious candidates for the mechanism of superconductivity in the cuprates. As HgBa<sub>2</sub>CuO<sub>4+ $\delta$ </sub> and Tl<sub>2</sub>Ba<sub>2</sub>CuO<sub>6+ $\delta$ </sub> are concerned, our results are consistent with previous estimates [5–7], but our approach does not rely on a particular type of  $\lambda_c$  measurement and a model dependent estimate of this quantity, but more generally on determination of the anisotropy  $\gamma$ . This quantity can be deduced with rather high precision from magnetization [12], specific heat [18], magnetic torque [13], *etc.* measurements on bulk samples. To summarize, we have shown that models relying on interlayer coupling cannot be considered as candidates for the mechanism of superconductivity in cuprate superconductors. Indeed the interlayer contribution to the superfluid is very small. For this reason the materials can be viewed as a stack of weakly coupled superconducting slabs of finite thickness.

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